An Empirical Investigation of OpenMP Based Implementation of Simplex Algorithm

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Agenda

- Introduction
- Objectives
- Parallel Algorithm
- Experimental Results and Observations
- Conclusion

What is the Simplex Algorithm?

- The Simplex Algorithm is a widely used algorithm to solve optimization problems subject to a set of linear relationships.
- Linear problems usually involve scheduling, assignment, location, network flow, and associated domains which are often required in industries that deal with manufacturing, distribution, transportation and finance.
- The Standard Simplex Method refers to the original algorithm proposed by George Dantzig.

What is the Simplex Algorithm?

- The linear relations are represented in the form of a table and we iterate on the table finding entering and leaving variables with each iteration. It requires the previous **iteration** to be completed before the new solution can be computed, thus restricting the scope of parallelization.
- LP problems are usually modelled as :

$$\textit{maximize / minimize } \mathbf{Z} = \mathbf{CX}$$

$$\textit{subject to } \mathbf{AX} = \mathbf{B} \ (\mathbf{X >= 0})$$

where **A** : Constraint Matrix **B** : Constant Vector

C : Objective Function Coefficient Vector **X** : Solution Vector

What is the Simplex Algorithm?

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & \cdots & A_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ A_{m1} & A_{m2} & \cdots & \cdots & A_{mn} \end{bmatrix}_{mxn} \quad \begin{bmatrix} B_1 \\ \cdots \\ \cdots \\ B_m \end{bmatrix}_{mx1} \quad \begin{bmatrix} C_1 & C_2 & \cdots & \cdots & C_n \end{bmatrix}_{1xn} \quad \begin{bmatrix} X_1 \\ \cdots \\ \cdots \\ X_n \end{bmatrix}_{nx1}$$

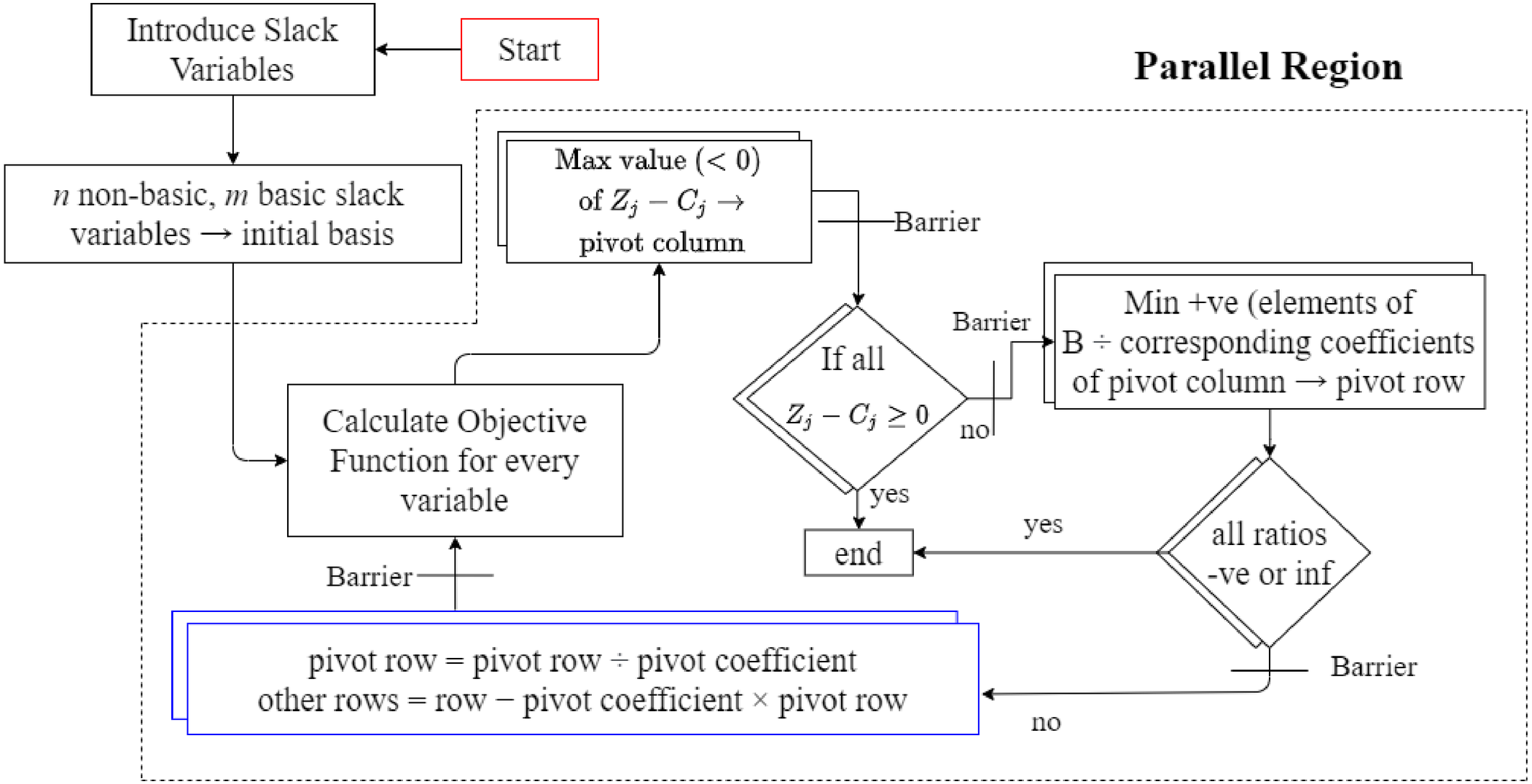
- Every row in the constraint matrix represent the coefficient of every variable for a given linear constraint.
- Every element in the constant vector corresponds to one linear constraint.
- Element in the Objective function coefficient vector corresponds to the coefficient of the variables in the objective function.
- Linear inequalities can be converted to equalities using slack and surplus variables.

Objectives

In this paper, we study:

- a shared-memory based parallel implementation of the standard simplex algorithm.
- the impact of the density of the underlying matrix on the overall performance.
- the **scalability** of the parallel algorithm over a range of matrix densities.
- the effect of increasing the number of constraints.
- the effect of varying the number of variables.

Simplex Algorithm



Parallel Algorithm

1. **Slack variables** are introduced to convert inequality constraint to equality constraint with **0 as their objective function coefficient**.
2. A tableau is made with **slack variables as the initial basis**. These 2 steps are done serially.
3. Threads are assigned to **individual columns** to calculate the value of the objective function and reduced costs**.**
4. **Maximum negative value** of reduced cost is found out using a custom class and reduction clause to get the **pivot column/ entering variable**. We terminate if not found.

Parallel Algorithm

5. Similarly, the **minimum row-wise ratio** is found out to get the pivot row/ leaving variable using reduction clause and custom-defined class. Unbounded and no solution condition is checked using no wait clause.
6. The **pivot coefficient** thus found is used to first update all the values in the pivot row. The remaining rows are updated using pragma for construct and within each row, columns use **SIMD vectorization.**
7. Local variables are updated by a single thread. **Repeat again from step 3** till the termination condition is satisfied.

Implementation Details

1. **Block Structured Notation**
   For efficient storage and modification fo the matrix elements we have made use of the block structure notation proposed by Dantzig:

$$\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix}$$

2. **Class-based key-value pair** to store maximum/minimum value along with its index
3. **Optimal Scheduling clause and Load Balancing**
   Guided scheduling was used to tackle the load balancing problem
4. **Optimal SIMD Units**
   4 was found to be the optimal *simdlen()*

Pseudo Code


```
# pragma omp parallel <shared variables>
{
//Step 3 & 4
# pragma omp for <schedule> <reduction>
for(int j=0;j<C;j++){
   find max negative value (max_value)
   from reduced cost row
}

Corresponding index=max_index
& column=Pivot Col

#pragma omp single nowait
max_value=0
```

Parallel region starts

Scheduling and reduction clauses

```
do{
    // Step 5
    #pragma omp for <schedule><reduction>
    for(int j=0;j<R;j++) {
        find the min value
        (min_value) from Min Ratio column
        do count++ for negative values
    }

    #pragma omp single nowait
    {
        if count == R
            there is unbounded/no solution
            flag=False break
        else
           Row corresponding to
           min_value is the
           Pivot row with index= min_index
```

Single thread with nowait clause to check for terminating condition

Pseudo Code


```
// Step 6
pivot = table[min_index][max_index]
#pragma omp barrier

#pragma omp parallel for
for(int i=0;i<C;i++)
    update Pivot Row
#pragma omp parallel for
for(int i=0;i<R;i++)
    #pragma omp simd
    for(int i=0;i<C;i++)
         update all elements except
         that of Pivot row
}
```

barrier Construct

simd Construct

```
//Step 3 & 4 repeated
    #pragma omp for <schedule> <reduction>
        for(int i=0;i<C;i++)
          find the new reduced cost values
          for updated table
          countNegative++ for negative values

    #pragma omp single
         Update the initial conditions
    }while(countNegative and flag)
}
```

Single thread to update variables

Machine Architecture

Specifications	Bench 1	Bench 2
Model Name	Silver 4214R CPU @ 2.40GHz	CPU E5-2640 v3 @ 2.60GHz
Core(s) per socket	12	8
Socket(s)	2	2
L1d Cache	768KB	32 KB
L2 Cache	24 MB	0.265 MB
L3 Cache	33 MB	20.48 MB
GNU GCC Version	9.3.0	10.2.0

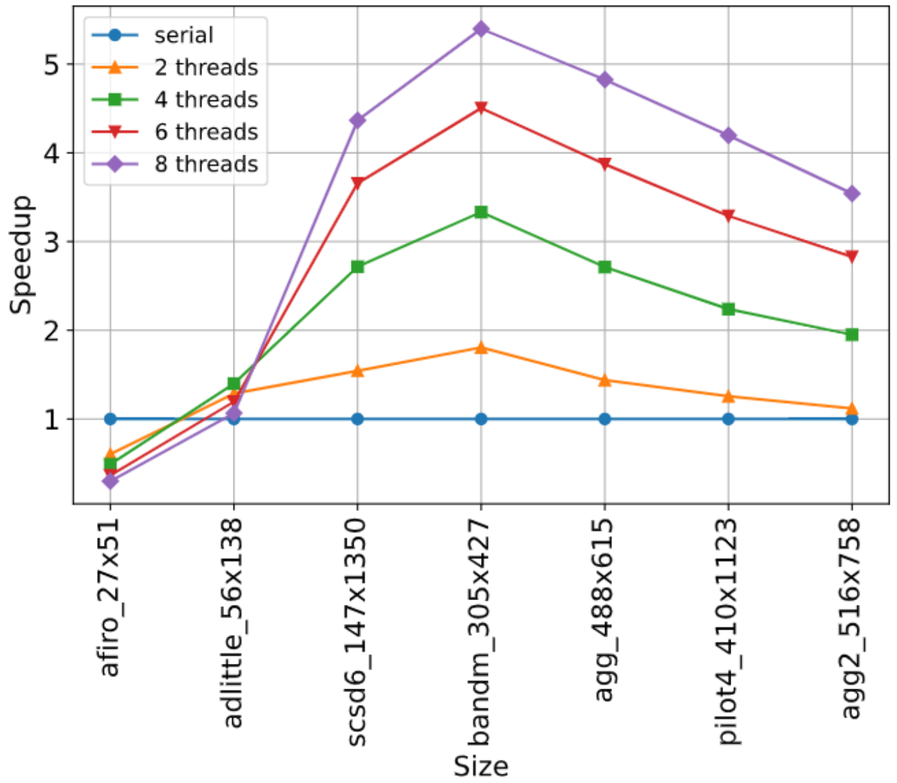
Datasets

- Netlib datasets [1]
- Computationally generated dataset with curated size and density
  - 256 constraints with varying number of variables with 0.5 density
  - 256 variables with varying number of constraints with 0.5 density
  - 512 variables and 512 constraints with varying density

1: http://www.netlib.org/

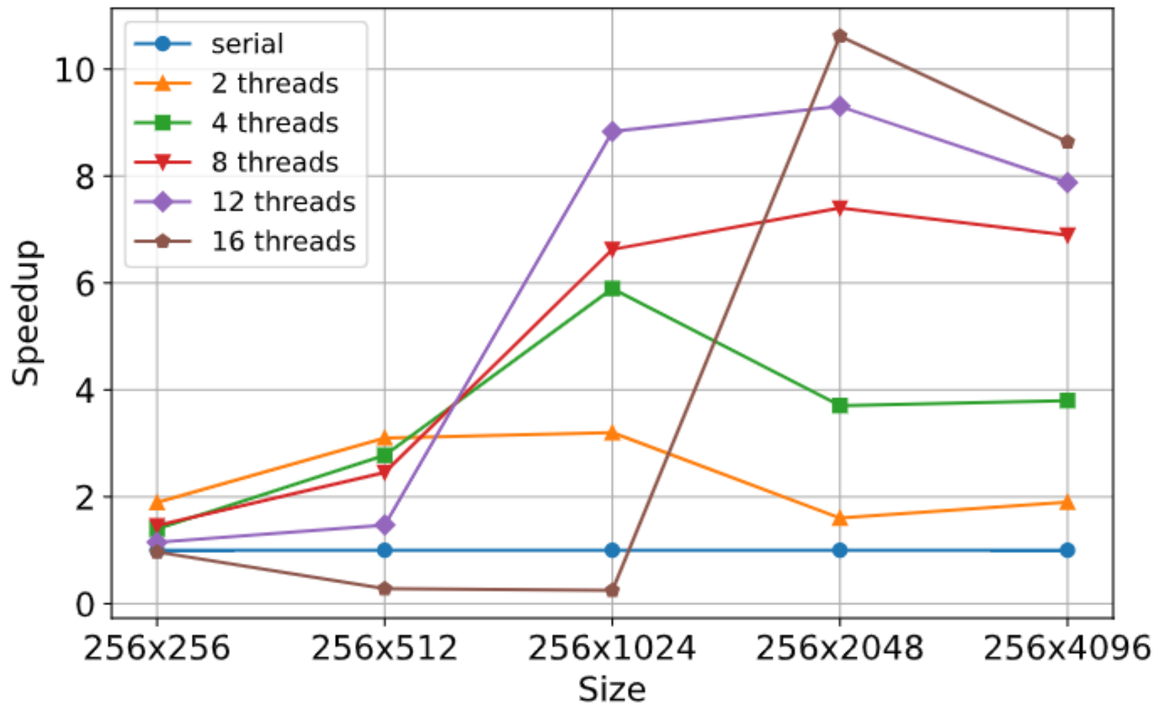
NETLIB Dataset Analysis

- Netlib dataset is the standard dataset for LP problems.
- For smaller problem sizes parallel algorithm does not lead to an improvement in performance due to thread overheads.
- The speedup for all the datasets remained within the linear upper bound.



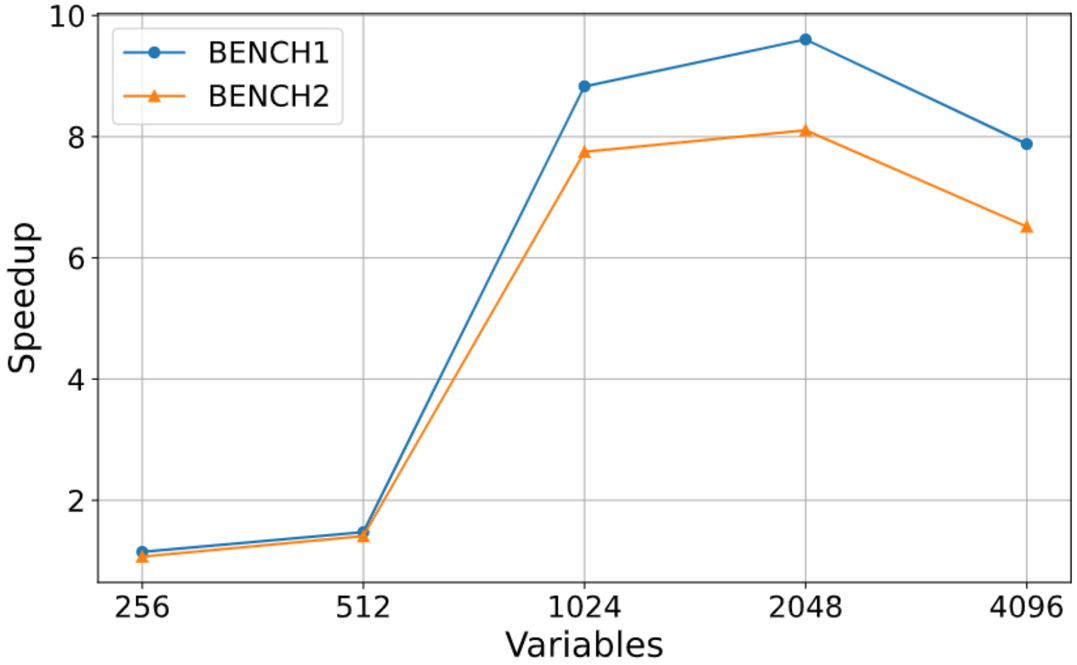
Variation of the Number of Variables

- it is observed that the speedup for each thread size increases (up to a point at mid-size) and then decreases. Peak values for large thread counts occur at larger problem sizes.
- At the largest problem size, the reduction in the peak performance is greatest for small thread counts (1, 2 and 4) and smallest for the larger thread counts (8,12 and 16)
- We could observe super-linear speedup in the case of 2 and 4 threads for certain problem sizes.



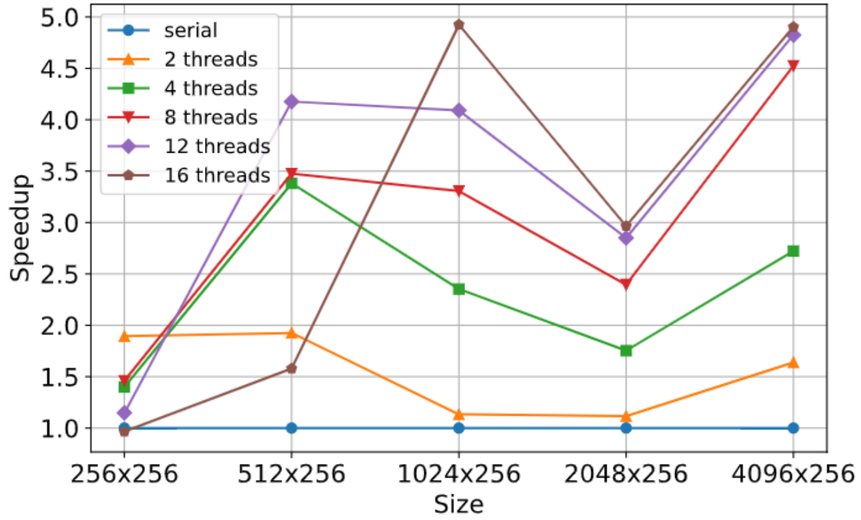
Variation of the Number of Variables

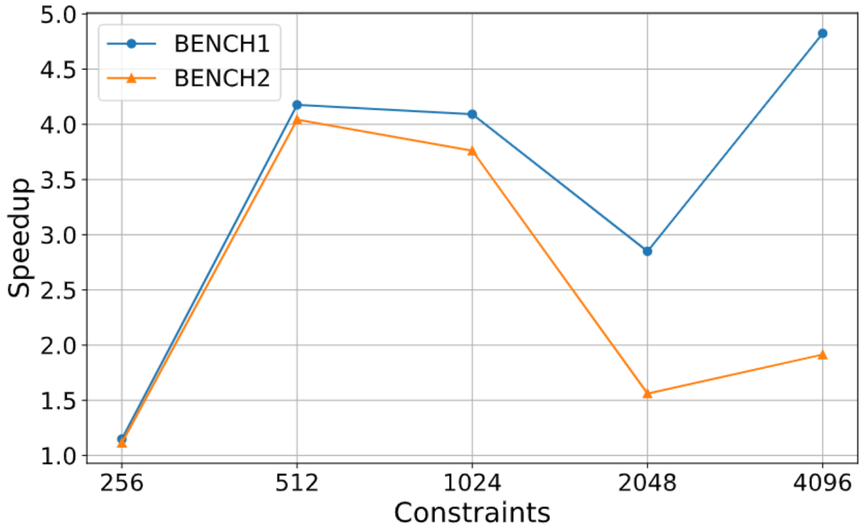
- The speedup achieved in the BENCH2 is similar to the one achieved in the BENCH1, till the problem size fits in the L1 and L2 cache of the respective systems.
- In the case of BENCH2, when the work allocated per thread exceeds the L2 cache size and results in data being continuously fetched from the L3 cache
- Our implementation observes a maximum speedup of 10.2 with 16 threads for 256 × 2048 which is comparable to the maximum speedup for a problem size of 256 constraints in the state-of-the-art implementation



Variation of the Number of Constraints

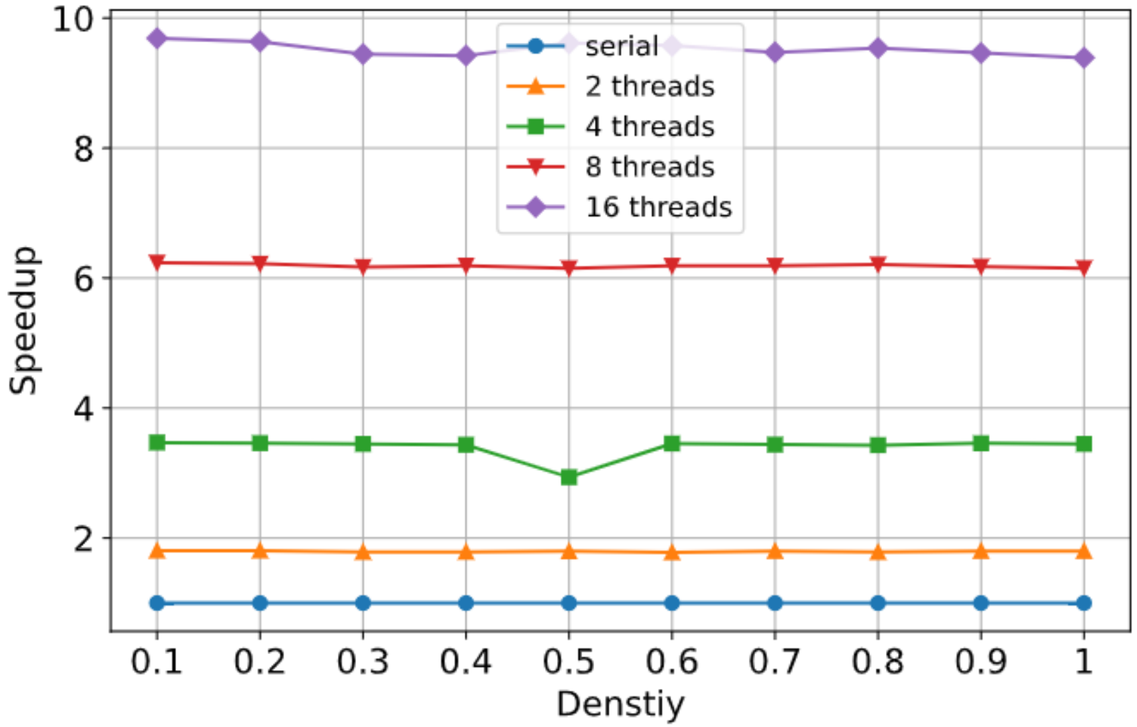
- On increasing the constraints for 256 variables, the speedup increases faster , as compared to when the number of variables increased.
- For 2048 constraints with 256 variables the problem size exceeds the L3 cache limit for the BENCH1. Thus we observe a drop.
- The speedup increases for larger problem size as threads increase to 16 and vice versa is seen for smaller problem sizes.





Variation in Matrix Density

- We have considered 512x512 matrices with densities varying from 0.1 to 1 in steps of 0.1.
- The speedup in all the cases remains almost constant or increases a little when the density of the matrices increases.
- Hence, the parallel algorithm is scalable in that nature.

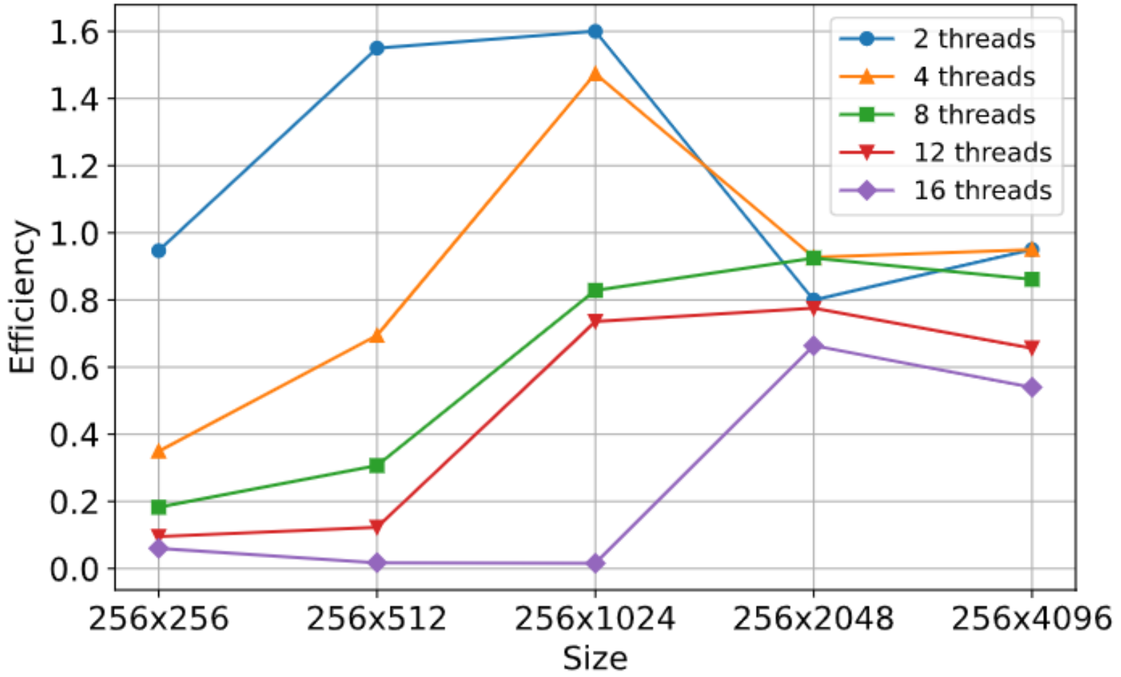


Key Observations

- Using more threads increases the synchronization overhead, while using a lower number of threads reduces the parallelization. The optimal limit on the number of threads lies in between the two extremes.
- There exists a critical problem size for each thread where the nature of the speedup changes from increasing to decreasing on either side of that critical number. This critical value is achieved at a larger problem size when using a larger thread count.
- Smaller problems performed better with a lower number of threads while larger problems perform better with a higher number of threads.

Key Observations

- Efficiency decreases with an increase in the number of threads mainly due to **synchronization overhead**.
- With the increase in problem size, **cache fulfillment** will also contribute significantly to the time needed to fetch the data leading to a drop in performance metrics



Conclusions

Vectorization	Number of constraints dominate more than number of variables	Scalable Algorithm
• Contributes significantly towards improving the performance • Constrained by the hardware properties of the system as well as the problem structure.	• The number of computations increases more with the increase in the number of constraints in comparison to the number of variables	• Our parallel algorithm proved to be fairly scalable, in terms of relative speedup, for the matrices of varying densities (in range of 0.1 to 1)

Questions ?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